

Table A.1 Continuous distributions

Distribution	Notation	Parameters
Uniform	$\theta \sim U(a, b)$ $p(\theta) = U(\theta a, b)$	boundaries a, b with $b > a$
Normal	$\theta \sim N(\mu, \sigma^2)$ $p(\theta) = N(\theta \mu, \sigma^2)$	location μ scale $\sigma > 0$
Multivariate normal	$\theta \sim N(\mu, \Sigma)$ $p(\theta) = N(\theta \mu, \Sigma)$ (implicit dimension d)	symmetric, pos. def., $d \times d$ cov. matrix Σ
Gamma	$\theta \sim \text{Gamma}(\alpha, \beta)$ $p(\theta) = \text{Gamma}(\theta \alpha, \beta)$	shape $\alpha > 0$ inverse scale $\beta > 0$
Inverse-gamma	$\theta \sim \text{Inv-gamma}(\alpha, \beta)$ $p(\theta) = \text{Inv-gamma}(\theta \alpha, \beta)$	shape $\alpha > 0$ scale $\beta > 0$
Chi-square	$\theta \sim \chi_\nu^2$ $p(\theta) = \chi_\nu^2(\theta)$	deg. of freedom $\nu > 0$
Inverse-chi-square	$\theta \sim \text{Inv-}\chi_\nu^2$ $p(\theta) = \text{Inv-}\chi_\nu^2(\theta)$	deg. of freedom $\nu > 0$
Scaled inverse-chi-square	$\theta \sim \text{Inv-}\chi_\nu^2(\nu, s^2)$ $p(\theta) = \text{Inv-}\chi_\nu^2(\theta \nu, s^2)$	deg. of freedom $\nu > 0$ scale $s > 0$
Exponential	$\theta \sim \text{Expon}(\beta)$ $p(\theta) = \text{Expon}(\theta \beta)$	inverse scale $\beta > 0$
Wishart	$W \sim \text{Wishart}_\nu(S)$ $p(W) = \text{Wishart}_\nu(W S)$ (implicit dimension $k \times k$)	deg. of freedom ν symmetric, pos. def. $k \times k$ scale matrix S
Inverse-Wishart	$W \sim \text{Inv-Wishart}_\nu(S^{-1})$ $p(W) = \text{Inv-Wishart}_\nu(W S^{-1})$ (implicit dimension $k \times k$)	deg. of freedom ν symmetric, pos. def. $k \times k$ scale matrix S

Density function	Mean, variance, and mode
$p(\theta) = \frac{1}{b-a}, \theta \in [a, b]$	$E(\theta) = \frac{a+b}{2}, \text{var}(\theta) = \frac{(b-a)^2}{12}$ no mode
$p(\theta) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(\theta - \mu)^2\right)$	$E(\theta) = \mu, \text{var}(\theta) = \sigma^2$ mode(θ) = μ
$p(\theta) = (2\pi)^{-d/2} \Sigma ^{-1/2} \times \exp\left(-\frac{1}{2}(\theta - \mu)^T \Sigma^{-1}(\theta - \mu)\right)$	$E(\theta) = \mu, \text{var}(\theta) = \Sigma$ mode(θ) = μ
$p(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta\theta}, \theta > 0$	$E(\theta) = \frac{\alpha}{\beta}$ $\text{var}(\theta) = \frac{\alpha}{\beta^2}$ mode(θ) = $\frac{\alpha-1}{\beta}$, for $\alpha \geq 1$
$p(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{-(\alpha+1)} e^{-\beta/\theta}, \theta > 0$	$E(\theta) = \frac{\beta}{\alpha-1}$, for $\alpha > 1$ $\text{var}(\theta) = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}$, $\alpha > 2$ mode(θ) = $\frac{\beta}{\alpha+1}$
$p(\theta) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)} \theta^{\nu/2-1} e^{-\theta/2}, \theta > 0$ same as $\text{Gamma}(\alpha = \frac{\nu}{2}, \beta = \frac{1}{2})$	$E(\theta) = \nu, \text{var}(\theta) = 2\nu$ mode(θ) = $\nu - 2$, for $\nu \geq 2$
$p(\theta) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)} \theta^{-(\nu/2+1)} e^{-1/(2\theta)}, \theta > 0$ same as $\text{Inv-gamma}(\alpha = \frac{\nu}{2}, \beta = \frac{1}{2})$	$E(\theta) = \frac{1}{\nu-2}$, for $\nu > 2$ $\text{var}(\theta) = \frac{2}{(\nu-2)^2(\nu-4)}$, $\nu > 4$ mode(θ) = $\frac{1}{\nu+2}$
$p(\theta) = \frac{(\nu/2)^{\nu/2}}{\Gamma(\nu/2)} s^\nu \theta^{-(\nu/2+1)} e^{-\nu s^2/(2\theta)}, \theta > 0$ same as $\text{Inv-gamma}(\alpha = \frac{\nu}{2}, \beta = \frac{\nu}{2} s^2)$	$E(\theta) = \frac{\nu}{\nu-2} s^2$ $\text{var}(\theta) = \frac{2\nu^2}{(\nu-2)^2(\nu-4)} s^4$ mode(θ) = $\frac{\nu}{\nu+2} s^2$
$p(\theta) = \beta e^{-\beta\theta}, \theta > 0$ same as $\text{Gamma}(\alpha = 1, \beta)$	$E(\theta) = \frac{1}{\beta}, \text{var}(\theta) = \frac{1}{\beta^2}$ mode(θ) = 0
$p(W) = \left(2^{\nu k/2} \pi^{k(k-1)/4} \prod_{i=1}^k \Gamma\left(\frac{\nu+1-i}{2}\right)\right)^{-1} \times S ^{-\nu/2} W ^{-(\nu-k-1)/2} \times \exp\left(-\frac{1}{2} \text{tr}(S^{-1}W)\right), W \text{ pos. def.}$	$E(W) = \nu S$
$p(W) = \left(2^{\nu k/2} \pi^{k(k-1)/4} \prod_{i=1}^k \Gamma\left(\frac{\nu+1-i}{2}\right)\right)^{-1} \times S ^{\nu/2} W ^{-(\nu+k+1)/2} \times \exp\left(-\frac{1}{2} \text{tr}(SW^{-1})\right), W \text{ pos. def.}$	$E(W) = (\nu - k - 1)^{-1} S$

Table A.1 Continuous distributions *continued*

Distribution	Notation	Parameters
Student- <i>t</i>	$\theta \sim t_\nu(\mu, \sigma^2)$ $p(\theta) = t_\nu(\theta \mu, \sigma^2)$ t_ν is short for $t_\nu(0, 1)$	deg. of freedom $\nu > 0$ location μ scale $\sigma > 0$
Multivariate Student- <i>t</i>	$\theta \sim t_\nu(\mu, \Sigma)$ $p(\theta) = t_\nu(\theta \mu, \Sigma)$ (implicit dimension d)	deg. of freedom $\nu > 0$ location $\mu = (\mu_1, \dots, \mu_d)$ symmetric, pos. def. $d \times d$ scale matrix Σ
Beta	$\theta \sim \text{Beta}(\alpha, \beta)$ $p(\theta) = \text{Beta}(\theta \alpha, \beta)$	'prior sample sizes' $\alpha > 0, \beta > 0$
Dirichlet	$\theta \sim \text{Dirichlet}(\alpha_1, \dots, \alpha_k)$ $p(\theta) = \text{Dirichlet}(\theta \alpha_1, \dots, \alpha_k)$	'prior sample sizes' $\alpha_j > 0; \alpha_0 \equiv \sum_{j=1}^k \alpha_j$

Table A.2 Discrete distributions

Distribution	Notation	Parameters
Poisson	$\theta \sim \text{Poisson}(\lambda)$ $p(\theta) = \text{Poisson}(\theta \lambda)$	'rate' $\lambda > 0$
Binomial	$\theta \sim \text{Bin}(n, p)$ $p(\theta) = \text{Bin}(\theta n, p)$	'sample size' n (pos. integer) 'probability' $p \in [0, 1]$
Multinomial	$\theta \sim \text{Multin}(n; p_1, \dots, p_k)$ $p(\theta) = \text{Multin}(\theta n; p_1, \dots, p_k)$	'sample size' n (pos. integer) 'probabilities' $p_j \in [0, 1];$ $\sum_{j=1}^k p_j = 1$
Negative binomial	$\theta \sim \text{Neg-bin}(\alpha, \beta)$ $p(\theta) = \text{Neg-bin}(\theta \alpha, \beta)$	shape $\alpha > 0$ inverse scale $\beta > 0$
Beta-binomial	$\theta \sim \text{Beta-bin}(n, \alpha, \beta)$ $p(\theta) = \text{Beta-bin}(\theta n, \alpha, \beta)$	'sample size' n (pos. integer) 'prior sample sizes' $\alpha > 0, \beta > 0$

Density function	Mean, variance, and mode
$p(\theta) = \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)\sqrt{\nu\pi}\sigma} (1 + \frac{1}{\nu}(\frac{\theta-\mu}{\sigma})^2)^{-(\nu+1)/2}$	$E(\theta) = \mu$, for $\nu > 1$ $\text{var}(\theta) = \frac{\nu}{\nu-2}\sigma^2$, for $\nu > 2$ $\text{mode}(\theta) = \mu$
$p(\theta) = \frac{\Gamma((\nu+d)/2)}{\Gamma(\nu/2)\nu^{d/2}\pi^{d/2} \Sigma ^{-1/2}} \times (1 + \frac{1}{\nu}(\theta - \mu)^T \Sigma^{-1}(\theta - \mu))^{-(\nu+d)/2}$	$E(\theta) = \mu$, for $\nu > 1$ $\text{var}(\theta) = \frac{\nu}{\nu-2}\Sigma$, for $\nu > 2$ $\text{mode}(\theta) = \mu$
$p(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1}$ $\theta \in [0, 1]$	$E(\theta) = \frac{\alpha}{\alpha+\beta}$ $\text{var}(\theta) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ $\text{mode}(\theta) = \frac{\alpha-1}{\alpha+\beta-2}$
$p(\theta) = \frac{\Gamma(\alpha_1+\dots+\alpha_k)}{\Gamma(\alpha_1)\dots\Gamma(\alpha_k)} \theta_1^{\alpha_1-1} \dots \theta_k^{\alpha_k-1}$ $\theta_1, \dots, \theta_k \geq 0; \sum_{j=1}^k \theta_j = 1$	$E(\theta_j) = \frac{\alpha_j}{\alpha_0}$ $\text{var}(\theta_j) = \frac{\alpha_j(\alpha_0-\alpha_j)}{\alpha_0^2(\alpha_0+1)}$ $\text{cov}(\theta_i, \theta_j) = -\frac{\alpha_i\alpha_j}{\alpha_0^2(\alpha_0+1)}$ $\text{mode}(\theta_j) = \frac{\alpha_j-1}{\alpha_0-k}$

Density function	Mean, variance, and mode
$p(\theta) = \frac{1}{\theta!} \lambda^\theta \exp(-\lambda)$ $\theta = 0, 1, 2, \dots$	$E(\theta) = \lambda$, $\text{var}(\theta) = \lambda$ $\text{mode}(\theta) = \lfloor \lambda \rfloor$
$p(\theta) = \binom{n}{\theta} p^\theta (1-p)^{n-\theta}$ $\theta = 0, 1, 2, \dots, n$	$E(\theta) = np$ $\text{var}(\theta) = np(1-p)$ $\text{mode}(\theta) = \lfloor (n+1)p \rfloor$
$p(\theta) = \binom{n}{\theta_1, \theta_2, \dots, \theta_k} p_1^{\theta_1} \dots p_k^{\theta_k}$ $\theta = 0, 1, 2, \dots, n; \sum_{j=1}^k \theta_j = n$	$E(\theta_j) = np_j$ $\text{var}(\theta_j) = np_j(1-p_j)$ $\text{cov}(\theta_i, \theta_j) = -np_i p_j$
$p(\theta) = \binom{\theta+\alpha-1}{-\alpha-1} \left(\frac{\beta}{\beta+1}\right)^\alpha \left(\frac{1}{\beta+1}\right)^\theta$ $\theta = 0, 1, 2, \dots$	$E(\theta) = \frac{\alpha}{\beta}$ $\text{var}(\theta) = \frac{\alpha}{\beta^2}(\beta+1)$
$p(\theta) = \frac{\Gamma(n+1)}{\Gamma(\theta+1)\Gamma(n-\theta+1)} \frac{\Gamma(a+\theta)\Gamma(n+b-\theta)}{\Gamma(a+b+n)}$ $\times \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}$, $\theta = 0, 1, 2, \dots, n$	$E(\theta) = n \frac{\alpha}{\alpha+\beta}$ $\text{var}(\theta) = n \frac{\alpha\beta(\alpha+\beta+n)}{(\alpha+\beta)^2(\alpha+\beta+1)}$